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# On the Flavour Dependence of the Mixed Quark-Gluon Condensate

Ketino Aladashvili[*] and Murman Margvelashvili[†]

*High Energy Physics Institute, Tbilisi State University*
*380086, Tbilisi, Georgia*

## Abstract

The flavour dependence of the mixed quark-gluon condensate is studied through the analysis of correlators of the hybrid current $a_\mu = g\bar{s}\gamma_\rho\gamma_5 G_{\rho\mu}d$. The flavour symmetry breaking for this type of condensates is found to be less than that for the quark condensates. For the ratio of strange to nonstrange condensates we obtain $R = 0.95 \pm 0.15$. For the kaon coupling to the current $a_\mu$ we find $\delta'^2 = (0.020 \pm 0.005)GeV^2$, which is an order of magnitude smaller than analogous chirally unsuppressed coupling.

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[*]E-mail: alada@bre.ge
[†]E-mail: mm@bre.ge, murman@cbdec1.cern.ch

# Introduction

The mixed quark gluon condensate $\langle 0|\bar{s}\sigma_{\mu\nu}G_{\mu\nu}s|0\rangle$ is an important phenomenological parameter characterizing the structure of QCD vacuum. Considerable attention has been paid in the past to determination of it's value, as well as of the ratio

$$R = \frac{\langle \bar{s}\sigma_{\mu\nu}G_{\mu\nu}s\rangle}{\langle \bar{d}\sigma_{\mu\nu}G_{\mu\nu}d\rangle} \tag{1}$$

of strange and nonstrange quark-gluon condensates [1-5].

A QCD sum rule (SR) analysis of the baryon decuplet indicates the value $R \simeq 1.3$ [4] while various hybrid sum rules favour the values in the lower range $R = 0.5 - 0.85$[1, 2, 3]. At the same time, the analysis of heavy to light mesons [5] results in the restriction $1 - R < 0.15$.

The two point functions of the hybrid current $a_\mu = g\bar{s}\gamma_\rho\gamma_5 G_{\rho\mu}d$ which have been studied in [1, 2] by the QCD sum rules technique are potentially the most sensitive to the value of $R$. The reason is, that in this case the leading contributions come from the mixed quark-gluon condensates of interest and the correlators are themselves proportional to the $SU(3)$ breaking. In the present work we continue the study of the correlators of the hybrid current $a_\mu$ [1, 2], being motivated by several reasons:

First, we are interested in the value of the kaon coupling to the hybrid current

$$\langle 0|a_\mu|K(p)\rangle = -f_K\delta'^2 p_\mu$$

which was left out in the the previous analyses. This parameter measures the Chiral symmetry breaking for kaon (in the chiral limit $\delta'^2 = 0$ [6]). Besides, this quantity can contribute significantly to various kaon transitions calculated in the Vertex Sum Rules approach (see e.g. [7]).

Next, we have calculated some additional contributions to theoretical expressions which happen to be important numerically and affect the value of the strange quark-gluon condensate. The parameter set which we use also differs from that of previous estimates.

For the determination of the unknown qantities we apply the recently proposed procedure of simultaneous analysis of several sum rules [8] which we believe provides the means of more detailed study for the type of problems considered here. The allowed range of unknown parameters is determined through the requirement of the compatibility of different SRs which is tested using the $\chi^2$ criterium.

# Equations

Consider the correlation functions of $a_\mu$ with pseudoscalar and axial vector currents:

$$ig \int dx e^{iqx} \langle 0|T\left(\bar{d}\gamma_\mu\gamma_5 s(x), \bar{s}\gamma_\rho\gamma_5 G_{\rho\nu}d(0)\right)|0\rangle = g_{\mu\nu}\Pi_1^{ax}(q^2) + q_\mu q_\nu \Pi_2^{ax}(q^2) \tag{2}$$

$$ig \int dx e^{iqx} \langle 0|T\left(\bar{d}\gamma_5 s(x), \bar{s}\gamma_\rho\gamma_5 G_{\rho\mu}d(0)\right)|0\rangle = q_\mu \Pi^{ps}(q^2) \tag{3}$$
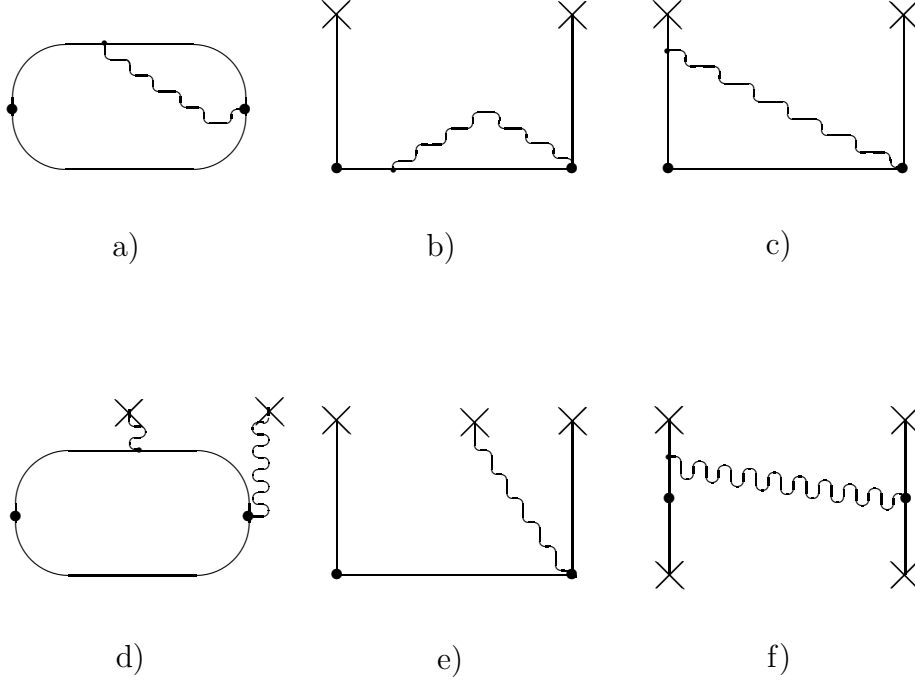
Fig.1

The operator product expansion for the invariant functions in (2), (3) is obtained in the Euclidean region by calculating diagrams shown in fig.1 and has the following form:

$$\Pi^{ps}(-Q^2) = \frac{\langle \overline{d}\sigma G d \rangle - \langle \overline{s}\sigma G s \rangle}{4Q^2} + \frac{\alpha_s}{48\pi^3}Q^2\left(ln^2(Q^2/\mu^2) - ln(Q^2/\mu^2)\right)$$

$$-\frac{\alpha_s}{3\pi}\left(\langle \overline{d}d \rangle - \langle \overline{s}s \rangle\right)ln(Q^2/\mu^2) + \frac{m_s\langle \alpha_s/\pi G^2 \rangle}{8Q^2}\left(ln(Q^2/\mu^2) - 1\right)$$

$$+\frac{4\pi\alpha_s m_s}{27}\frac{\left(3\langle \overline{d}d \rangle^2 + \langle \overline{s}s \rangle^2 - 9\langle \overline{d}d \rangle\langle \overline{s}s \rangle\right)\rho}{Q^4}$$

$$-\frac{\pi^2\left(\langle \overline{d}d \rangle - \langle \overline{s}s \rangle\right)\langle \frac{\alpha_s}{\pi}G^2 \rangle}{9\quad\quad Q^4} \tag{4}$$

$$\Pi_2^{ax}(-Q^2) = -\frac{m_s\langle \overline{s}\sigma G s \rangle}{6Q^4} - \frac{2\alpha_s m_s}{9\pi Q^2}\left(5\langle \overline{s}s \rangle - (ln(Q^2/\mu^2) + \frac{1}{3})\langle \overline{d}d \rangle\right)$$

$$+\frac{8\pi\alpha_s}{27}\frac{\left(\langle \overline{s}s \rangle^2 - \langle \overline{d}d \rangle^2\right)\rho}{Q^4} \tag{5}$$

$$\Pi_1^{ax}(-Q^2) = -\frac{m_s\langle \overline{d}\sigma G d \rangle}{4Q^2} + \frac{m_s\langle \overline{s}\sigma G s \rangle}{12Q^2}$$

$$-\frac{\alpha_s}{3\pi}m_s\left(\langle \overline{s}s \rangle - \frac{5}{3}\langle \overline{d}d \rangle\right)ln(Q^2/\mu^2) + \frac{8\pi\alpha_s}{27}\frac{\left(\langle \overline{s}s \rangle^2 - \langle \overline{d}d \rangle^2\right)\rho}{Q^2} \tag{6}$$

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Here we work in the chiral limit $m_d = 0$ for the $d$ quark and restrict ourselves to the terms linear in $SU(3)$ breaking. The calculation has been performed in the $\overline{MS}$ scheme and the fixed point gauge technique has been used for condensate contributions.

The quark and gluon condensate contributions are given by the one loop diagrams b),c) and d). The latter one requires the removal of infrared divergence which occurs due to the mixing of the operators $m_s G^2$ and $ig\overline{s}Gs$. In spite of their one loop suppression, the quark and gluon condensate contributions are quite important and comparable to that of the quark-gluon condensates. Inclusion of these terms shifts $R$ to its higher values, but mostly affects $\delta'^2$ in comparison with [2].

In (4-6) the four quark condensates are reduced to the squares of quark condensates, but we retain a factor $\rho$ to account for the violation of vacuum dominance. The tree level dimension seven contributions have been evaluated in the vacuum saturation approximation. However, due to numerical smallness of these terms the approximation has only minor effect on our final results.

We keep the perturbative contribution (diagram a) ) only for $\Pi^{ps}$ where it is of the order $O(m_s)$ , for $\Pi^{ax}_i$ the similar contribution is proportional to $m_s^2$ and thus negligible in our approximation. The gluon condensate does not contribute to equations (5), (6) to the considered order, since it is also $O(m_s^2)$ [2].

The anomalous dimensions of the operators are accounted for by taking e.g.

$$\langle\bar{q}\sigma Gq\rangle(\mu) = \langle\bar{q}\sigma Gq\rangle(1GeV)\left(\frac{\alpha_s(1GeV)}{\alpha_s(\mu)}\right)^{31/54} ; \quad m_s = m_s(1GeV)\left(\frac{\alpha_s(1GeV)}{\alpha_s(\mu)}\right)^{-4/9} \quad (7)$$

We saturate the phenomenological sides of SRs by the lowest lying intermediate states: the $K$ meson for the pseudoscalar correlator, and the $K$ and $K_1$ for the axial vector one. We use the usual model spectra with $K$ and $K_1$ as narrow resonances and the continuum, equal to the theoretical one, starting at some threshold $s_0$. Following [2], instead of the two nearby resonances $K_1(1270)$ and $K_1(1400)$ we substitute an effective resonance $K_1(1335)$.

After the Borel transformation [9] we obtain the following set of equations:

$$\frac{(1-R)\langle\overline{d}\sigma Gd\rangle}{4M^2} + \frac{\alpha_s}{3\pi}\left(\langle\overline{d}d\rangle - \langle\overline{s}s\rangle\right)(1-e^{-x_0})$$

$$-\frac{\alpha_s}{48\pi^3}M^4\int_0^{x_0}e^{-x}x(1-2lnx)dx + \frac{m_s\langle\frac{\alpha_s}{\pi}G^2\rangle}{8M^2}\left(\int_0^{x_0}\frac{1}{x}(1-e^{-x})dx - 1\right)$$

$$+\frac{4\pi\alpha_s m_s}{27M^4}\rho\left(3\langle\overline{d}d\rangle^2 + \langle\overline{s}s\rangle^2 - 9\langle\overline{d}d\rangle\langle\overline{s}s\rangle\right) - \frac{\pi^2}{9M^4}\left(\langle\overline{d}d\rangle - \langle\overline{s}s\rangle\right)\langle\frac{\alpha_s}{\pi}G^2\rangle =$$

$$-\frac{f_K^2 m_K^2}{m_s M^2}\delta'^2 e^{-m_K^2/M^2} \quad (8)$$

$$-\frac{m_s R\langle\overline{d}\sigma Gd\rangle}{6M^4} - \frac{2\alpha_s m_s}{9\pi M^2}\left(5\langle\overline{s}s\rangle - \left(\int_0^{x_0}\frac{1}{x}(1-e^{-x})dx + \frac{1}{3}\right)\langle\overline{d}d\rangle\right)$$

$$+\frac{8\pi\alpha_s}{27M^4}\rho\left(\langle\overline{s}s\rangle^2 - \langle\overline{d}d\rangle^2\right) = \frac{f_K^2\delta'^2}{M^2}e^{-m_K^2/M^2} + \frac{C}{m_{K_1^2}M^2}e^{-m_{K_1}^2/M^2} \quad (9)$$

$$\frac{m_s(R-3)\langle\overline{d}\sigma Gd\rangle}{12M^2} + \frac{\alpha_s}{3\pi}m_s\left(\langle\overline{s}s\rangle - \frac{5}{3}\langle\overline{d}d\rangle\right)(1-e^{-x_0})$$
$$+\frac{8\pi\alpha_s}{27M^4}\rho\left(\langle\overline{s}s\rangle^2 - \langle\overline{d}d\rangle^2\right) = -\frac{C}{M^2}e^{-m_{K_1}^2/M^2} \tag{10}$$

where $C$ is the residue of the $K_1$ effective pole, and $x_0 = \frac{s_0}{M^2}$ with $s_0$ being the continuum onset. The running parameters are taken at $\mu^2 = M^2$.

Eqs.(8-10) constitute a system of sum rules to be used for the determination of the un-known quantities $R$, $\delta'^2$ and $C$. Following the previous analyses [1, 2] we could eliminate the couplings $C$ and $\delta'^2$ from these equations and express $R$ as a function of QCD param-eters and $M^2$. This however would mean that the two of three equations are required to hold exactly at any $M^2$, while the third one is considered as approximate equation with all uncertainties and errors accumulated in it. Instead, we follow the procedure developed in [8] and consider the Borel transformed SRs (8-10) as a system of independent approx-imate equations, where we try to estimate the expected errors and find a set of unknown parameters which makes the whole system maximally consistent.

The error analysis of such a system is the most problematic task where the model assumptions are unavoidable. Following [8] we model the errors of equations (8-10) in the following way: For each one of these equations we choose some reference point $\tilde{M}_i^2$ (i=8,9,10) of the Borel parameter, where the *relative* error $w_i$ is assumed to be minimal. Then the *absolute* error distribution is described by a simple function

$$D(M^2) = \begin{cases} \frac{\tilde{M}^2 - M_0^2}{M^2 - M_0^2} & \text{if } M^2 < \tilde{M}^2 \ GeV^2 \\ 1 & \text{if } M^2 \geq \tilde{M}^2 \ GeV^2 \end{cases} \tag{11}$$

which has a pole at $M_0^2 < \tilde{M}^2$ (to account for the divergence of the power series) and is constant for $M^2 > \tilde{M}^2$.

Finally the *absolute* error is calculated as

$$\Delta_i(M^2) = D(M^2)w_i A_i \tag{12}$$

where $A_i \equiv A_i(\tilde{M}^2)$ denote the r.h.s.'s of eqs.(8-10) at the reference values of the Borel parameter. Note, that due to the decrease of $A_i(M^2)$ at high $M^2$ eq.(12) corresponds to growth of the *relative* errors for both high and low values of this parameter.

To get an idea of the scale of expected errors, we first examine each of the equations (8-10) separately and check their stability with respect to $M^2$ for different values of $\langle\overline{s}Gs\rangle$. Fig.2 shows the $M^2$ dependence of $\delta'^2$ and $C$ (normalized to their final values) as defined from eqs.(8-10) with $R = 0.85$. The SR (8) is the most sensitive to the value of $R$. It becomes more stable for higher values of this parameter, but as can be seen from the figure, even in this case the stability is not very good, which can be attributed to the unknown higher orders corrections or the roughness of the model of the phenomenological spectrum. On the other hand eq.(9) is the most stable in $M^2$ for a wide range of $R$ and we consider it to be the most reliable one. For eq.(10) the stability plateau starts at higher $M^2$ values than for eqs.(9-10), which can be considered as indication of the higher characteristic mass scale for this equation ($m_{K_1}^2 \simeq 1.8 GeV^2$). Thus our check shows that we should expect a greater relative error in (8) than in other equations and use higher $M^2$ values for SR (10).

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# Results and Discussion

In order to fix the unknown quantities $R$, $\delta'^2$ and $C$ we first scan all their possible values. For each set we evaluate the equations (8-10) at four separated $M^2$ points, within the range of their expected validity. Thus we build up a system of 12 linear approximate equations where for each one we calculate the errors according to eqs. (11), (12). We look for a set of unknown parameters which makes these 12 equations maximally consistent, e.i. gives minimal $\chi^2_{d.o.f}$. Once we find such a solution, we test its stability against variations of different parameters involved in the derivation ($M^2$ points, $\omega_i$ etc.) and adjust the latter to get the least sensitivite result. Note, that this is not a standard $\chi^2$ procedure since in our case the errors depend on the solutions themselves. The details and justification of the approach can be found in [8], where we have used the similar method to restrict the values of standard condensates.

For eqs.(8),(9) we take the $M^2$ points to be in the range $(0.8 \div 2)\ GeV^2$, the effective pole in the error distribution is set to $M_0 = 0.7 GeV^2$ and we use $\tilde{M}^2 = 1.2 GeV^2$ as a reference value of $M^2$. For eq.(11) we increase all these values by $(0.2 - 0.4) GeV^2$. The typical relative errors which we have used in our analysis are: $\omega_8 = 0.25$, $\omega_9 = 0.15$ and $\omega_{10} = 0.20$. This is quite a conservative choice, especially taking into account that the relative errors grow for both higher and lower values of $M^2$. The continuum onset in equations (9),(10) is taken as $s_0 \simeq 2.5 GeV^2$, while in the equation (8) where there is no $K_1$ contribution, the continuum threshold is taken to be lower, $s_0 \simeq 1.5 GeV^2$.

For the numeric evaluation we use the following values of the input parameters:

$$\Lambda_{MS} = 150 MeV, \quad m_s = 180 MeV[11,\ 12],$$
$$\langle \bar{q}q \rangle = -(0.24 \text{Gev})^3, \quad \langle \bar{s}s \rangle = (0.7 \pm 0.1)\langle \bar{d}d \rangle[10], \tag{13}$$
$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 1.2 \cdot 10^{-2} GeV^4[9], \quad \langle \bar{d}Gd \rangle / \langle \bar{d}d \rangle = 0.63 GeV^2, \quad \rho = 4[8].^1$$

The minimal $\chi^2$ solution is given by $R = 0.93$, $\delta'^2 = 0.021 GeV^2$ and $C = -6 \cdot 10^{-4} GeV^4$ corresponding to $\chi^2_{min} = 0.6$. This solution is quite stable with respect to the variations of different parameters introduced in our procedure. The changes of $M^2$ points by $\pm 0.2 GeV^2$, of $w_i$s by 20% and $M_0$ by $\pm 0.1 GeV^2$ do not affect the result significantly. We have also checked the dependence on the parameters of eq.(13). So, changing $\langle \bar{s}s \rangle / \langle \bar{d}d \rangle = 0.6(0.8)$ we get $R = 0.94(0.89)$, $\delta'^2 = 0.02(0.023)$; $m_s = 160(200) MeV$ gives $R = 0.92(0.95)$, $\delta'^2 = 0.019(0.024) GeV^2$ and $M_0^2 = 0.8(0.5) GeV^2$ gives $R = 0.91(0.96)$ and $\delta'^2 = 0.024(0.019) GeV^2$ respectively. The final result is also insensitive to the variations of $s_0$ and $s_0'$.

In fig.3 we have plotted the ranges in the $(R, \delta'^2)$ plane corresponding to the solutions with $\chi^2_{d.o.f} < 1$ and $\chi^2_{d.o.f} < \chi^2_{min} + 1 = 1.6$. We consider all the solutions having $\chi^2_{d.o.f} < \chi^2_{min} + 1 = 1.6$ as acceptable and thus quote our final result as

$$R = 0.95 \pm 0.15 \quad \delta'^2 = (0.020 \pm 0.005) GeV^2 \tag{14}$$

From the figure one can notice that the correlation of these quantities is insignificant.

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[1]This corresponds to the results of [8]where instead of the present value we have used $\langle \sqrt{\alpha_s} \bar{q}q \rangle = -(0.24 \text{Gev})^3$ of [9].

The value of $R$ is larger than that in previous estimates [1-3] and is consistent with the symmetry limit $R = 1$. The errors are also bigger, but to our feeling here they look more realistic. The result conforms well with the conjecture of [5], based on the analysis of heavy-to-light systems, that inclusion of the gluon field into the operator reduces the strength of the flavor symmetry violation.

The parameter $\delta'^2$ is an order of magnitude smaller than the similar, chirally unsuppressed coupling of the kaon to another hybrid current $\bar{s}\gamma_\nu \tilde{G}_{\nu\mu} d$ ($\delta^2 \sim 0.2 GeV^2$ [6],[3]). Consequently, it will not contribute significantly to the vertex sum rules except the cases proportional to the $SU(3)$ flavour symmetry breaking.

# Acnowledgements

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# Figure captions

**Fig.2** Stability test for SRs (8-10) with $R = 0.85$:

$\bullet$ — $\delta'^2/(0.002 GeV^2)$ from eq.(8);

$\times$ — $\delta'^2/(0.002 GeV^2)$ from eq.(10) with $C = 6 \cdot 10^{-4} GeV^4$;

$+$ — $C/(-6 \cdot 10^{-4} GeV^4)$ determined from eq.(9).

**Fig.3** The allowed region in the plane $(R, \delta'^2)$. The areas with $\chi^2_{d.o.f} < 1$ and $\chi^2_{d.o.f} < \chi^2_{min} + 1$ are indicated.